**Supporting information: S1**

Estimating species occupancy across multiple sampling seasons with temporal autologistic occupancy models via the autoOcc R package

<Author information has been removed for the review process, it will be added in the event the manuscript is accepted for publication>

## The autologistic occupancy model is a simplification of the dynamic occupancy model

The autologistic occupancy model is a special case of the dynamic occupancy model. To show this I will explain the standard formulation of the dynamic model and then share the assumptions that must be made to reduce the dynamic model to an autologistic parameterization. As the detection part of the model is identical to a standard dynamic occupancy model, and is already explained within the manuscript itself, I will skip over it in this supporting information.

Thus, for *i* in 1,…,*I* sites and *t* in 1,…,*T* primary sampling periods (hereafter seasons), let *zi,t*be the latent binary occupancy status of a species at site *i* and time *t*. During the first season we have no knowledge of the occupancy status before sampling began. Thus, we estimate occupancy in the first time step, which we refer to as the initial occupancy probability *ψ*.

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|  |  | Eq. S1 |

Where *ψ* can be made a function of covariates using the logit link. In Eq. S1, *a0* represents the intercept, is a vector of slope terms, and ***qi*** a conformable vector of covariates to be multiplied by each respective slope term.

Following the first season we condition the occupancy status, *zi,t,*on the occupancy status in the previous season, *zi,t-1*. Doing so allows us to separately estimate local colonization (γi) and extinction (εi) probabilities. Thus, for *t>1*, the latent state model is

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|  |  | Eq. S2 |

Thus, the occupancy status in the previous timestep serves as an indicator variable. If the species is present in the previous timestep then the probability in Eq. S2 simplifies to , which is also known as the persistence probability, φi. Conversely, if the species is not present, then Eq. S2 simplifies to estimating local colonization, γi. As with Eq. S1, both γi and extinction εi can be made a function of covariates. For γi, *b0* is the colonization intercept, a vector of slope terms, and ***xi*** a conformable vector of covariates. Likewise, for , *c0* is the colonization intercept, a vector of slope terms, and ***yi*** a conformable vector of covariates. Note that while I only index the covariates by site, the model does allow for temporal or spatiotemporal covariates, which would just require adding a *t* subscript to those covariate vectors.

Eq. S1 and S2 represent the entire latent state of the standard dynamic occupancy model. Furthermore, unlike the autologistic model, Eq. S2 represents a means parameterization as it models the absolute levels of the logit-scale colonization and extinction probabilities. For an autologistic model, Eq. S1 is unchanged, we only modify the dynamic equation of S2. First, we replace in Eq. S2 with it’s complement, φi so that the equation becomes.

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|  |  | Eq. S3 |

Where *d0* is the persistence intercept, is a vector of slope terms, and ***yi*** is a conformable vector of covariates. At this stage, we have the pieces in place to construct the autologistic term in our model. First, we need to change the model to move from a means parameterization to a difference parameterization. Thus, let our autologistic term ϑ = . In other words, ϑ represents the difference between the colonization intercept and the persistence intercept, hence the term difference parameterization. Second, we assume that covariates affect colonization and persistence in identical ways. As such,  and =. In other words, we use the same covariates for colonization and persistence. With a few algebraic manipulations the model moves from it’s standard dynamic parameterization to an autologistic model

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|  | The standard parameterizationMultiply through  Factor the terms with | Eq. S$ |

Inputting the logit-linear predictors into and canceling out the additional and terms results in a logit-linear predictor that is essentially identical to the autologistic parameterization of the manuscript (Eq. S2), . To make it fully identical, the intercept would need to be included within the vector of slope terms and a 1 would be added to the start of the covariate vector.